ANALYSIS OF THE HYPERBOLIC PROCESS OF HEAT CONDUCTION FOR A HOLLOW CYLINDER HEATED BY A MOVING SOURCE

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The hyperbolic equation of heat conduction is solved for a hollow cylinder heated by a movable source. A number of characteristics resulting from the hyperbolic nature of the process of heat transfer are critically analyzed.

The problem of determining the temperature field of a hollow finite cooled cylinder, which is heated by a source moving according to an arbitrary law, has been investigated in [1]. However, the description based on the parabolic equation may be inadequate if the rate of displacement of the sources and also the rates of change of other thermophysical quantities become comparable with the square of the velocity of heat propagation

$$w^2 = \frac{a}{\tau}, \tag{1}$$

which may occur, for example, during the displacement of an electrical arc in the operating chamber of a plasmatron [2, 3].

We consider a problem analogous to [1] but take into consideration the hyperbolic nature of the rapid thermal process.

Eliminating the vector **q** from the basic law of heat conduction [4], written in the form

$$\mathbf{q} = -\lambda \operatorname{grad} T - \tau \frac{d\mathbf{q}}{dt}, \tag{2}$$

where d/dt is the total time derivative, and the heat balance equation

$$W - c\rho \frac{dT}{dt} = \operatorname{div} \mathbf{q} \tag{3}$$

for **v** = const we obtain the following equation for heat conduction:

$$\frac{1}{w^2} \frac{d^2T}{dt^2} + \frac{1}{a} \frac{dT}{dt} = \Delta T + \frac{1}{\lambda} W + \frac{\tau}{\lambda} \frac{dW}{dt}, \tag{4}$$

where the operator d^2/dt^2 is given by

$$\frac{d^2}{dt^2} = \frac{\partial^2}{\partial t^2} + 2 \left(\mathbf{v} \operatorname{grad}\right) \frac{\partial}{\partial t} + (\mathbf{v} \operatorname{grad}) (\mathbf{v} \operatorname{grad}). \tag{5}$$

Thus the heat balance is affected by not only the intensity of the source but also by the total change of this intensity in time:

$$\frac{dW}{dt} = \frac{\partial W}{\partial t} + (\mathbf{v} \operatorname{grad} W). \tag{6}$$

If this quantity is large, the last term in Eq. (4) cannot be discarded.

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In accordance with the above discussion the hyperbolic generalization of the problem of [1] is written (for $\mathbf{v} = 0$) in the following form:

$$\frac{1}{\mathrm{Pe}_\tau^2}\frac{\partial^2\theta}{\partial \mathrm{Fo}^2}+\frac{\partial\theta}{\partial \mathrm{Fo}}=\frac{\partial^2\theta}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\theta}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2\theta}{\partial\varphi^2}+(k\pi)^2\frac{\partial^2\theta}{\partial\eta^2}-\mathrm{Po}\left(f+\frac{1}{\mathrm{Pe}_\tau^2}\frac{\partial f}{\partial \mathrm{Fo}}\right), \tag{7}$$

$$\theta(\rho,\ \varphi,\ \eta,\ 0)=1, \tag{8}$$

$$\frac{\partial\theta(\rho,\ \varphi,\ \eta,\ 0)}{\partial \mathrm{Fo}}=0, \tag{9}$$

$$\left(\frac{\partial\theta}{\partial\rho}-\mathrm{Bi}_1\theta\right)_{\rho=1}=0, \tag{10}$$

$$\left(\frac{\partial\theta}{\partial\rho}+\frac{1}{\rho_0}\mathrm{Bi}_2\theta\right)_{\rho=\rho_0}=\frac{1}{\rho_0}\mathrm{Bi}_2, \tag{11}$$

$$\theta(\rho,\ \varphi,\ \eta,\ \mathrm{Fo})=\theta(\rho,\ \varphi+2\pi,\ \eta,\ \mathrm{Fo}), \tag{12}$$

$$\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}=\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=\pi}=0, \tag{13}$$

where $\mathrm{Pe}_\tau = wR_1/a$ is the dimensionless velocity of propagation of thermal perturbation.

The choice of the function $f(\rho,\ \varphi,\ \eta,\ \mathrm{Fo})$ is governed by the trajectory, rate, and the space–time distribution of the heat flux. For example, if

$$f(\rho,\ \varphi,\ \eta,\ \mathrm{Fo})=\delta(\rho-1)\sum_{l=1}^{n}u_l(\mathrm{Fo})\,\delta(\eta-\eta_l), \tag{14}$$

where $\{\eta_l\}$ is an arbitrary sequence of values of the axial coordinate η ($0 < \eta_l < \pi$, $l = \overline{1, n}$) and $u_l(\mathrm{Fo})$ is some finite function with support $[\mathrm{Fo}_l,\ \mathrm{Fo}_{l+1})$, then (14) describes a thin high-intensity ring source of variable intensity oriented toward the inner surface and moving along the axis of the cylinder in steps.

If

$$f(\rho,\ \varphi,\ \eta,\ \mathrm{Fo})=\frac{1}{(2\pi)^{3/2}\sigma}\exp\left[-\frac{(\eta-\mathrm{Pe}_v\mathrm{Fo})^2}{2\sigma^2}\right]\vartheta\left(\frac{\varphi-\mathrm{Pe}_\omega\mathrm{Fo}}{2\pi},\ \frac{\sigma^2}{2\pi}\right)\delta(\rho-\rho_0), \tag{15}$$

where ϑ is Jacobi's theta function, σ^2 is the variance, and ($\mathrm{Pe_v Fo}$, $\mathrm{Pe_w Fo}$) is the movable center of the distribution, then we obtain a description of the source with normal distribution of the heat flux density describing a spiral line along the outer surface of the cylinder and terminating its existence at the instant $\mathrm{Fo} = 2\pi/\mathrm{Pe_v}$ and so forth.

We shall seek the solution of Eq. (7) with conditions (8)-(13) by the method of finite integral transforms. The triple transform [5]

$$\bar\theta(\mu,\ m,\ q,\ \mathrm{Fo})=\int_0^{2\pi}\int_1^{\rho_0}\int_0^{\pi}\exp(im\varphi)\,\rho V_m(\mu\rho)\cos q\eta\,\theta(\rho,\ \varphi,\ \eta,\ \mathrm{Fo})\,d\varphi d\rho d\eta \tag{16}$$

satisfies the following Cauchy problem:

$$\frac{1}{\mathrm{Pe}_\tau^2}\frac{d^2\bar\theta}{d\mathrm{Fo}^2}+\frac{d\bar\theta}{d\mathrm{Fo}}=-(\mu^2+k^2\pi^2q^2)\,\bar\theta+2\pi^2\mathrm{Bi}_2V_0(\mu\rho_0)\,\delta_{0m}\delta_{0q}-\mathrm{Po}\left(\bar f+\frac{1}{\mathrm{Pe}_\tau^2}\frac{d\bar f}{d\mathrm{Fo}}\right), \tag{17}$$

$$\bar\theta(\mu,\ m,\ q,\ 0)=\frac{2\pi^2}{\mu^2}\left[\mathrm{Bi}_2V_0(\mu\rho_0)+\frac{2}{\pi}\mathrm{Bi}_1\right]\delta_{0m}\delta_{0q}, \tag{18}$$

$$\frac{d\bar\theta(\mu,\ m,\ q,\ 0)}{d\mathrm{Fo}}=0, \tag{19}$$

where δ_{ik} is the Kronecker symbol and $\bar f = \bar f(\mu, m, q, \mathrm{Fo})$ is the transform of the function $f(\rho,\ \varphi,\ \eta,\ \mathrm{Fo})$ of type (16).

The generalization for the triple transform (16) has the form [5]

$$\theta(\rho,\ \varphi,\ \eta,\ \mathrm{Fo})=\frac{1}{4}\sum_{m=-\infty}^{\infty}\sum_{\mu}\sum_{q=-\infty}^{\infty}\mu^2A_{\mu m}V_m(\mu\rho)\exp(-im\varphi)\cos q\eta\,\bar\theta(\mu,\ m,\ q,\ \mathrm{Fo}), \tag{20}$$

where $\{\mu^2\}$ are the eigenvalues of the corresponding Sturm–Liouville operator, a part of which is given in [1], and $\{V_m(\mu\rho)\}$ are the eigenfunctions which are simultaneously the kernel of the Hankel part of transform (16). The quantity $A_{\mu m}$ is given by

$$A_{\mu m} = \left[\frac{\pi^2}{4}(\mu^2\rho_0^2 + \mathrm{Bi}_2^2 - m^2)\, V_m^2(\mu\rho_0) - (\mu^2 + \mathrm{Bi}_1^2 - m^2)\right]^{-1}. \tag{21}$$

The expression for $V_m(\mu\rho)$ and also some asymptotic forms are given in [1].

The solution of problem (17)-(19), obtained with the use of Laplace transforms, has the following form:

$$\begin{aligned}\bar{\theta}(\mu, m, q, \mathrm{Fo}) = \frac{\mathrm{Po}}{\Delta_0}\int_0^{\mathrm{Fo}} \left\{\left(1 + \frac{z_2}{\mathrm{Pe}_\tau^2}\right)\exp\left[z_2(\mathrm{Fo}-\lambda)\right] - \left(1 + \frac{z_1}{\mathrm{Pe}_\tau^2}\right)\right. \\ \left.\times \exp\left[z_1(\mathrm{Fo}-\lambda)\right]\right\} \bar{f}(\mu, m, q, \lambda)\, d\lambda + \frac{1}{\Delta_0 \mathrm{Pe}_\tau^2}\left[\mathrm{Po}\,\bar{f}(\mu, m, q, 0)\right. \\ \left. - \frac{4\pi \mathrm{Bi}_1}{\mu^2} z_2 \delta_{0m}\delta_{0q}\right]\exp(z_1 \mathrm{Fo}) - \frac{1}{\Delta_0 \mathrm{Pe}_\tau^2}\left[\mathrm{Po}\,\bar{f}(\mu, m, q, 0)\right. \\ \left. - \frac{4\pi \mathrm{Bi}_1}{\mu^2} z_1 \delta_{0m}\delta_{0q}\right]\exp(z_2 \mathrm{Fo}) + \frac{2\pi^2 \mathrm{Bi}_2 V_0(\mu\rho_0)}{\mu^2}\delta_{0m}\delta_{0q},\end{aligned} \tag{22}$$

where z_1 and z_2 are the roots of the equation

$$\frac{1}{\mathrm{Pe}_\tau^2} z^2 + z + (\mu^2 + k^2\pi^2 q^2) = 0 \tag{23}$$

and

$$\Delta_0 = \sqrt{1 - \frac{4}{\mathrm{Pe}_\tau^2}(\mu^2 + k^2\pi^2 q^2)}\,. \tag{24}$$

Substituting (22) into (20) we obtain the solution of problem (7)-(13) for an arbitrary source $f = f(\rho, \varphi, \eta, \mathrm{Fo})$:

$$\begin{aligned}\theta(\rho, \varphi, \eta, \mathrm{Fo}) = \frac{\frac{1}{\mathrm{Bi}_1} + \ln\rho}{\frac{1}{\mathrm{Bi}_1} + \frac{1}{\mathrm{Bi}_2} + \ln\rho_0} + \pi \mathrm{Bi}_1 \exp\left(-\frac{\mathrm{Pe}_\tau^2}{2}\mathrm{Fo}\right) \\ \times \sum_\mu A_{\mu 0} V_0(\mu\rho)\left[\mathrm{ch}\frac{\Delta_0' \mathrm{Pe}_\tau^2}{2}\mathrm{Fo} - \frac{1}{\Delta_0'}\mathrm{sh}\frac{\Delta_0' \mathrm{Pe}_\tau^2}{2}\mathrm{Fo}\right] + \frac{\mathrm{Po}}{2\mathrm{Pe}_\tau^2} \\ \times \exp\left(-\frac{\mathrm{Pe}_\tau^2}{2}\mathrm{Fo}\right)\sum_{m,\mu,q}\mu^2 A_{\mu m} V_m(\mu\rho)\exp(-im\varphi)\cos q\eta\,\frac{1}{\Delta_0} \\ \times \mathrm{sh}\frac{\Delta_0 \mathrm{Pe}_\tau^2}{2}\mathrm{Fo}\,\bar{f}(\mu, m, q, \mathrm{Fo}) - \frac{\mathrm{Po}}{4}\sum_{m,\mu,q}\mu^2 A_{\mu m} V_m(\mu\rho)\exp(-im\varphi) \\ \times \cos q\eta \int_0^{\mathrm{Fo}} \exp\left(-\frac{\mathrm{Pe}_\tau^2}{2}\lambda\right)\left[\mathrm{ch}\frac{\Delta_0 \mathrm{Pe}_\tau^2}{2}\lambda + \frac{1}{\Delta_0}\mathrm{sh}\frac{\Delta_0 \mathrm{Pe}_\tau^2}{2}\lambda\right]\bar{f}(\mu, m, q, \mathrm{Fo}-\lambda)\, d\lambda,\end{aligned} \tag{25}$$

where Δ_0' is equal to Δ_0 for q = 0 and the symbol $\sum\limits_{m,\mu,q}$ denotes triple summation of type (20).

Since for all finite values of μ and q and sufficiently large Pe_τ

$$\frac{\Delta_0 \mathrm{Pe}_\tau^2}{2} = \frac{\mathrm{Pe}_\tau^2}{2} - (\mu^2 + k^2\pi^2 q^2) - \frac{1}{\mathrm{Pe}_\tau^2}(\mu^2 + k^2\pi^2 q^2)^2 - \ldots, \tag{26}$$

for $\mathrm{Pe}_\tau \to \infty$ from (25) we get

$$\begin{aligned}\theta(\rho, \varphi, \eta, \mathrm{Fo}) = \frac{\frac{1}{\mathrm{Bi}_1} + \ln\rho}{\frac{1}{\mathrm{Bi}_1} + \frac{1}{\mathrm{Bi}_2} + \ln\rho_0} + \pi \mathrm{Bi}_1 \sum_\mu A_{\mu 0} V_0(\mu\rho) \\ \times \exp(-\mu^2 \mathrm{Fo}) - \frac{\mathrm{Po}}{4}\sum_{m,\mu,q}\mu^2 A_{\mu m} V_m(\mu\rho)\exp(-im\varphi)\cos q\eta \int_0^{\mathrm{Fo}} \exp\left[-(\mu^2 + k^2\pi^2 q^2)\lambda\right]\bar{f}(\mu, m, q, \mathrm{Fo}-\lambda)\, d\lambda,\end{aligned} \tag{27}$$

which coincides with the solution of the corresponding hyperbolic problem [1, 5].

If the source is absent, i.e., $f(\rho, \varphi, \eta, Fo) \equiv 0$, then only the "background" component

$$\theta_\Phi(\rho, Fo) = \frac{\frac{1}{Bi_1} + \ln\rho}{\frac{1}{Bi_1} + \frac{1}{Bi_2} + \ln\rho_0} + \pi Bi_1 \exp\left(-\frac{Pe_\tau^2}{2} Fo\right) \times \sum_\mu A_{\mu 0} V_0(\mu\rho)\left[\operatorname{ch}\frac{\Delta_0' Pe_\tau^2}{2} Fo + \frac{1}{\Delta_0'} \operatorname{sh}\frac{\Delta_0' Pe_\tau^2}{2} Fo\right]. \quad (28)$$

remains in (25).

Since the sequence $\{\mu_n^2\}$ increases without limit, for any finite Pe_T there is a number $n = N$, starting from which

$$1 - \frac{4}{Pe_\tau^2}\mu_n^2 \leqslant 0 \quad (n > N). \quad (29)$$

Accordingly, starting from this term the hyperbolic sine and cosine go over into circular. Hence the first $(N-1)$ terms of series (28) describe the aperiodic contribution to the heat transfer process and the remaining terms describe the periodic contribution. The first eigenfrequency of the thermal oscillations in a hollow cylinder is

$$\omega_N = \frac{Pe_\tau^2}{2}\sqrt{\frac{4}{Pe_\tau^2}\mu_N^2 - 1}. \quad (30)$$

If $Pe_T \gg 1$, then $N \gg 1$. In this case from the asymptotic representation [6]

$$\mu_n^2 \approx \frac{\pi^2 n^2}{(\rho_0 - 1)^2} \quad (n \gg 1) \quad (31)$$

we get

$$N \gtrsim \frac{\rho_0 - 1}{2\pi} Pe_\tau. \quad (32)$$

For example, for steel [7] $w \sim 1800$ m/sec; hence $Pe_T \sim 10^6$-10^7 and even for relatively thin walls ($\rho_0 \sim 1.05$) we have $N \sim 10^4$-10^5. But, since for large n [4] $A_{\mu 0} V_0(\mu\rho) \sim 1/(\rho_0 - 1)\mu_n^2$, the thermal wave appearing in the walls of a hollow steel cylinder and that corresponding to the first eigenfrequency goes into solution (28) with an initial amplitude of the order of 10^{-12}.

Thus, if $Pe_T \gg 1$, the "background" component (28) may be replaced by the corresponding part from (27) with an accuracy adequate for engineering practice.

Let us consider the complete solution (25).

If $Po \ll Pe_T^2$, the second term in (25) can be discarded. However, if $Po \sim Pe_T^2$ and $\bar{f}(\mu, m, q, 0) \neq 0$, then this series must be retained, i.e., under the influence of strong high-intensity sources the hyperbolic nature of heat conduction begins to show up.

Let the source be displaced. As an example we consider a thin high-intensity source placed at the inner wall oriented along the generatrix and rotating with angular speed Pe_ω:

$$f(\rho, \varphi, \eta, Fo) = \delta(\rho - 1)\,\delta_n(\varphi - Pe_\omega Fo), \quad (33)$$

where the last delta function is periodic with period $[0, 2\pi]$.

Then [5]

$$\bar{f}(\mu, m, q, Fo) = 2\exp(im Pe_\omega Fo)\,\delta_{0q}, \quad (34)$$

from which we get

$$\bar{f} + \frac{1}{Pe_\tau^2}\frac{\partial \bar{f}}{\partial Fo} = 2\left(1 + im\frac{Pe_\omega}{Pe_\tau^2}\right)\exp(im Pe_\omega Fo)\,\delta_{0q}. \quad (35)$$

Thus the last term in Eqs. (4) and (7) can be discarded for small angular speeds $Pe_\omega \ll Pe_T^2$. However, if $Pe_\omega \sim Pe_T^2$, then as seen from (35), the presence of the term $1/Pe_T^2\, \partial f/\partial Fo$ in the equation of heat conduction has a significant effect on the form of the solution and for $Pe_\omega \gg Pe_T^2$ it even begins to dominate the contribution due to f.

Substituting (34) into (25) we obtain the solution for source (33):

$$\theta(\rho, \varphi, \eta, \mathrm{Fo}) = \theta_\Phi(\rho, \mathrm{Fo}) + \frac{\mathrm{Po}}{\mathrm{Pe}_\tau^2} \exp\left(-\frac{\mathrm{Pe}_\tau^2}{2}\mathrm{Fo}\right)$$
$$\times \sum_{m,\mu} \mu^2 A_{\mu m} V_m(\mu\rho) \exp(-im\varphi) \frac{1}{\Delta_0'} \operatorname{sh} \frac{\Delta_0' \mathrm{Pe}_\tau^2}{2} \mathrm{Fo} - \frac{\mathrm{Po}}{2}$$
$$\times \sum_{m,\mu} \mu^2 A_{\mu m} V_m(\mu\rho) \exp(-im\varphi) \frac{1}{\Delta_0'} \int_0^{\mathrm{Fo}} \exp\left(-\frac{\mathrm{Pe}_\tau^2}{2}\lambda\right)$$
$$\times \left[\operatorname{sh} \frac{\Delta_0' \mathrm{Pe}_\tau^2}{2}\lambda + \Delta_0' \operatorname{ch} \frac{\Delta_0' \mathrm{Pe}_\tau^2}{2}\lambda\right] \exp\left[im\,\mathrm{Pe}_\omega(\mathrm{Fo} - \lambda)\right] d\lambda. \tag{36}$$

According to the Riemann–Lebesgue theorem [8] for $\mathrm{Pe}_\omega \to \infty$ the last integral vanishes for all m except m = 0. As easily seen, the solution thus obtained coincides with the solution of the analogous problem for $f(\rho, \varphi, \eta, \mathrm{Fo}) = (1/2\pi)\delta(\rho-1)$. Hence an infinitely rapid rotation of the source is equivalent to an uniformly distributed intensity $\mathrm{Po}/2\pi$.

For $\mathrm{Pe}_\omega \neq \infty$ the integral in (36) is equal to

$$\frac{1+\Delta_0'}{2} \frac{\exp(im\,\mathrm{Pe}_\omega \mathrm{Fo}) - \exp\left[-\frac{1-\Delta_0'}{2}\mathrm{Pe}_\tau^2 \mathrm{Fo}\right]}{\frac{1-\Delta_0'}{2}\mathrm{Pe}_\tau^2 + im\,\mathrm{Pe}_\omega} - \frac{1-\Delta_0'}{2} \frac{\exp(im\,\mathrm{Pe}_\omega \mathrm{Fo}) - \exp\left[-\frac{1+\Delta_0'}{2}\mathrm{Pe}_\tau^2 \mathrm{Fo}\right]}{\frac{1+\Delta_0'}{2}\mathrm{Pe}_\tau^2 + im\,\mathrm{Pe}_\omega}. \tag{37}$$

Investigating the behavior of the modulus of this expression we find that for angular speeds satisfying the relation

$$m\,\mathrm{Pe}_\omega = \pm \frac{\mathrm{Pe}_\tau^2}{2} \sqrt{\frac{4}{\mathrm{Pe}_\tau^2}\mu_{N+k}^2 - 2} \quad (m = 1, 2, \ldots;\ k = 0, 1, 2, \ldots), \tag{38}$$

the amplitude of the thermal oscillations acquires maxima, i.e., phenomena similar to resonance occur. The values given by (38) somewhat differ on the decreasing side from the corresponding frequencies of eigenoscillations (for example, at k = 0 from (30)), which is usual for oscillations in a medium with drag [9]. The ordinary parabolic mechanism of heat conduction obviously plays the role of such drag.

In conclusion we note another characteristic which follows from the basic law of heat conduction (2) and is of interest in the investigation of temperature fields appearing in the presence of rotating sources. Taking the curl of Eq. (2) and remembering that

$$\operatorname{curl} \operatorname{grad} T \equiv 0,$$

we obtain the equation for the vortex of the heat flux

$$\tau \frac{d \operatorname{curl} \mathbf{q}}{dt} + \operatorname{curl} \mathbf{q} = 0, \tag{39}$$

which shows that for $\tau \neq 0$ in general $\operatorname{curl}\mathbf{q} \neq 0$ also.

Therefore the vector field $\mathbf{q}$ ceases to be a potential field, as it would follow from Fourier's law in the classical case, and the possibility of formation of thermal vortices appears. From Stokes theorem [10]

$$\oint_L \frac{1}{\lambda} q_l\, dl = \frac{1}{\lambda} \int_S \operatorname{curl} \mathbf{q}\, ds \tag{40}$$

the circulation of the vector $(1/\lambda)\mathbf{q}$ along an arbitrary closed contour L is nonzero; hence an integral of type (40) (along a not necessarily closed trajectory) can no longer be interpreted as the total change of temperature in going from one isothermal surface to another.

Eliminating scalar T from system (2)-(3) we obtain the equation for the heat flux:

$$\frac{1}{w^2} \frac{d^2\mathbf{q}}{dt^2} + \frac{1}{a} \frac{d\mathbf{q}}{dt} = \Delta \mathbf{q} + \operatorname{curl} \operatorname{curl} \mathbf{q} - \operatorname{grad} W. \tag{41}$$

If $\tau \to 0$, then $w^2 \to \infty$, $\operatorname{curl}\operatorname{curl}\mathbf{q} \to 0$, and Eq. (41) goes over into the classical equation.

NOTATION

$\theta = (T_C - T)/(T_C - T_0)$;	
$\rho = r/R_1$;	
$\eta = \pi(z/2l)$	are the dimensionless temperature, and coordinates;
$Bi_1 = \alpha_1 R_1/\lambda$;	
$Bi_2 = \alpha_2 R_2/\lambda$;	
$Fo = at/R_1^2$;	
$Po = R_1^2 W_0/\lambda(T_C - T_0)$	are the Biot, Fourier, and Pomerantsev numbers respectively;
$\alpha_1, R_1, \alpha_2, R_2$	are the internal and external thermal conductivity coefficients and radii of the cylinder;
$c, \rho, \lambda, a, \mathbf{v}, t$	are the specific heat, density, thermal conductivity, thermal diffusivity, rate of convective transfer, and relaxation time;
W	is the distributed source intensity;
$Pe_\omega = R_1\omega/a$	is the dimensionless angular speed;
T_C, T_0	are the temperatures of the surrounding media;
$k = R_1/2l$	is the ratio of the radius of the cylinder to its length;
$\rho_0 = R_2/R_1$;	
Δ	is the Laplacian;
$\delta(z)$	is the delta function.

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